Calculation of isospin-mixing corrections for the isobaric analogue Fermi decay of ^{32}Cl

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As mentioned in Ref. [1], the branching ratio of the 1^+, $T = 1$ β decay of ^{32}Cl to the analogue 1^+, $T = 1$ state in ^{32}S provides a sensitive test of nuclear-struture-dependent isospin-symmetry-breaking (ISB) effects in superallowed Fermi β decays. The correction, denoted by δ_C, is defined by the equation $|M_F|^2 = |M_0|^2(1 - \delta_C)$, where M_F is the Fermi matrix element for the transition and M_0 is its value in the limit of strict isospin symmetry, which is broken by Coulomb and charge-dependent nuclear forces. We use a shell-model calculation of ISB for this case and compare to our experimentally observed value $\delta_C = 5.3(9)\%$ (see Ref. [1])

The technique is to introduce Coulomb and other charge-dependent terms into the shell-model Hamiltonian. However, because the Coulomb force is long range, the shell-model space has to be very large indeed to include all the potential states that the Coulomb interaction might connect. Currently this is not a practical proposition. To proceed, Towner and Hardy divide δ_C into two parts: δ_{C1} and δ_{C2}, where δ_{C1} arises from configuration mixing between states of the same spin in a shell-model calculation using a restricted basis (in this case the full *s, d* shell), while δ_{C2} separately encompasses mixing beyond this model space. Starting with δ_{C1}, we perform a shell-model calculation in the truncated $0\hbar\omega$ model space of the *s, d*-shell orbitals. Charge-dependent terms are added to the charge-independent Hamiltonians of USD, USDA, and USDB. The strengths of these charge-dependent terms are adjusted to reproduce the *b* and *c* coefficients of the isobaric multiplet mass equation as applied to the 1^+, $T = 1$ triplet of states in $A = 32$, the states involved in the β transition under study. The bulk of the isospin mixing in the 7001-keV IAS occurs with the nearby 1^+, $T = 0$ state at 7190 keV. In the limit of two-state mixing, perturbation theory implies that $\delta_{C1} \propto 1/(\Delta E)^2$, where ΔE is the energy separation of the analogue and non-analogue 1^+ states. Experimentally, it is known to be 188.2±1.2 keV (compared to the much larger 2–4 MeV of most $0^+ \rightarrow 0^+$ transitions). The shell model calculates this separation to be 184 keV with USD, 248 keV with USDA and 387 keV with USDB interactions. We avoid the large uncertainties this would impose on our calculation by following the Towner-Hardy recommendation [1] of scaling the calculated δ_{C1} value by a factor of $(\Delta E)^2_{\text{theo}}/(\Delta E)^2_{\text{exp}}$, the ratio of the square of the energy separation of the 1^+ states in the model calculation to that known experimentally. Following this procedure, the δ_{C1} values obtained in the three shell-model calculations are reasonably consistent: $\delta_{C1} = 3.73\%$ for USD, $\delta_{C1} = 3.32\%$ for USDA and $\delta_{C1} = 4.19\%$ for USDB. We average these three results and assign an uncertainty equal to half the spread between them to arrive at $\delta_{C1} = 3.75(45)\%$. As Fig. 1 shows, this is over an order of magnitude larger than δ_{C1} calculated for any of the 13 $0^+ \rightarrow 0^+$ transitions used to determine V_{ud}.

For the calculation of δ_{C2} we consider mixing with states outside the $0\hbar\omega$ shell-model space. The principal mixing is with states that have one more radial node. Such mixing effectively changes the radial function of the proton involved in the β decay relative to that of the neutron. The practical calculation, therefore, involves computing radial-overlap integrals with modeled proton and neutron radial functions. Details of how this is done are given in Ref. [2]. The radial functions are taken to be eigenfunctions of a Saxon-Woods potential whose strength is adjusted so that the asymptotic form of the radial function has

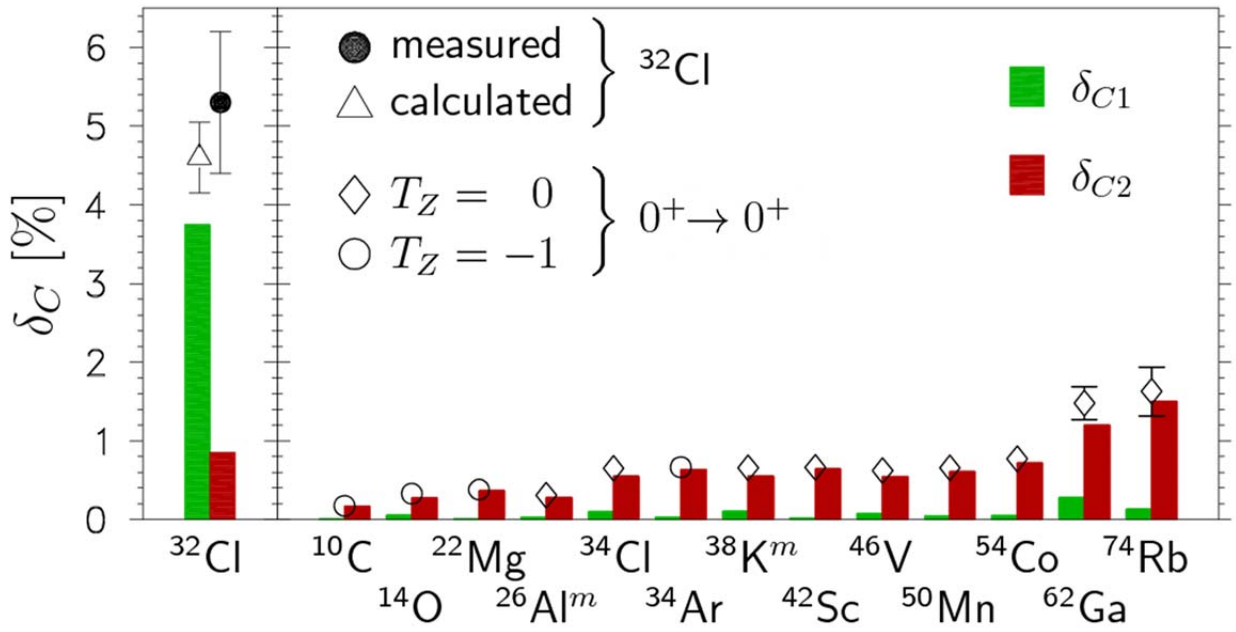

FIG. 1. Our determination of the isospin-breaking correction for ^{32}Cl (filled circle), and calculations for ^{32}Cl as well as other superallowed transitions (open points), with the δ_{C1} and δ_{C2} components shown separately. The measurement and prediction for ^{32}Cl, particularly the δ_{C1} component, is significantly larger than in any of the $0^+ \rightarrow 0^+$ transitions.

the correct dependence on the separation energy. The initial and final *A*-body states are expanded in a complete set of (*A* - 1)-parent states. The separation energies are the energy differences between the *A*-body state and the (*A* -1)-body parent states. A shell-model calculation is required to give the spectrum of parent states and the spectroscopic amplitudes of the expansion. For the three USD interactions, we compute δ_{C2} = 0.827% for USD and 0.865% for both USDA and USDB. Our adopted value is δ_{C2} = 0.85(3)%. The uncertainty, calculated in the same manner as described in Ref. [2], represents the range of results for the USD interactions, the different methodologies considered in adjusting the strength of the Saxon-Woods potential, and the uncertainty in the Saxon-Woods radius parameter as fitted to the experimental charge radius of ^{32}S.

Combining our adopted shell-model calculations, δ_{C1} = 3.75(45)% and δ_{C2} = 0.85(3)%, we find δ_C^{theor} = 4.6(5)%, which agrees with the experimentally determined 5.3(9)% of [1] within stated uncertainties. The agreement between theory and experiment in this case where δ_C is so large represents a very important validation of the theoretical procedures outlined here to calculate the ISB effects in nuclei. In particular, for (shell-model) calculations which separate configuration-mixing and radial-overlap components, this δ_{C1}-dominated result provides an especially sensitive benchmark for the approximations used when calculating configuration-mixing contributions to the total ISB effect in superallowed $0^+ \rightarrow 0^+$ decays.

This result has been recently published in Refs. [3, 4].

[1] J.C. Hardy, V.E. Iacob, D. Melconian, N. Nica, H.I. Park, G. Tabacaru, I.S. Towner, L. Trache, R.E. Tribble, and Y. Zhai, *Progress in Research*, Cyclotron Institute, Texas A&M University (2011-2012), p. I-16.

[2] I.S. Towner and J.C. Hardy, Phys. Rev. C **77**, 025501 (2008).

[3] D. Melconian, S. Triambak, C. Bordeanu, A. Garcia, J. Hardy, V. Iacob, N. Nica, H. Park, G. Tabacaru, I. Towner, L. Trache, R. Tribble and Y. Zhai, Phys. Rev. Letts. **107**, 182301 (2011).

[4] D. Melconian, S. Triambak, C. Bordeanu, A. Garcia, J. Hardy, V. Iacob, N. Nica, H. Park, G. Tabacaru, I. Towner, L. Trache, R. Tribble and Y. Zhai, Phy. Rev. C **85**, 025501 (2012).